TUTORIAL - 13

The following data gives the average life in hours and range in hours of 20 samples each of 5 lamps. Draw the control charts for mean and range and comment on the state of control.

Sample	1	2	3	4	5	6	7	8	9	10
Values of X	11.1	9.6	9.7	10.1	12.4	10.1	11.0	11.2	10.6	8.3
	9.4	10.8	10.0	8.4	10.0	10.2	11.5	10.0	14.4	10.2
	11.2	10.1	10.0	10.2	10.7	10.2	11.8	10.9	10.5	9.8
	10.4	10.8	9.8	9.4	10.1	11.2	11.0	10.2	10.5	9.5
	10.1	11.0	10.4	11.0	11.3	10.1	11.3	10.0	10.9	9.8
$\bar{X}_i$	10.44	10.46	9.98	9.82	10.9	10.36	11.32	10.86	11.38	9.52
R_i	1.8	1.4	0.7	2.6	2.4	1.1	0.8	1.2	3.9	1.9

Sample	11	12	13	14	15	16	17	18	19	20
Values of X	8.3	10.6	10.8	11.3	11.4	10.1	10.7	11.9	10.8	12.4
	10.2	9.9	10.2	11.4	11.2	10.1	12.8	11.9	12.1	11.1
	9.8	10.7	10.5	10.4	11.4	9.7	11.2	11.6	11.8	10.8
	9.5	10.2	8.4	10.6	10.1	9.8	11.2	12.4	9.4	11.0
	9.8	11.4	9.9	11.1	11.6	10.5	11.3	11.4	11.6	11.9
$\bar{X}_i$	9.52	10.56	9.96	10.96	11.14	10.04	11.44	11.84	11.14	11.44
R_i	1.9	1.5	2.4	1	1.5	0.8	2.1	1	2.7	1.6

$N = 20$, $n = 5$.

i) $\bar{\bar{X}} = \frac{\Sigma \bar{X}_i}{n} = \frac{213.08}{20} = 10.654 \approx 10.65$

ii) $R = \frac{\Sigma R_i}{n} = \frac{34.3}{20} = 1.715 \approx 1.72$

iii) Sample size $n = 5$, from statistics table

$A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.115$.

iv) To draw $\bar{x}$ - chart.

$CL = \bar{\bar{x}} = 10.65$

$UCL = \bar{\bar{x}} + A_2 \bar{R}$

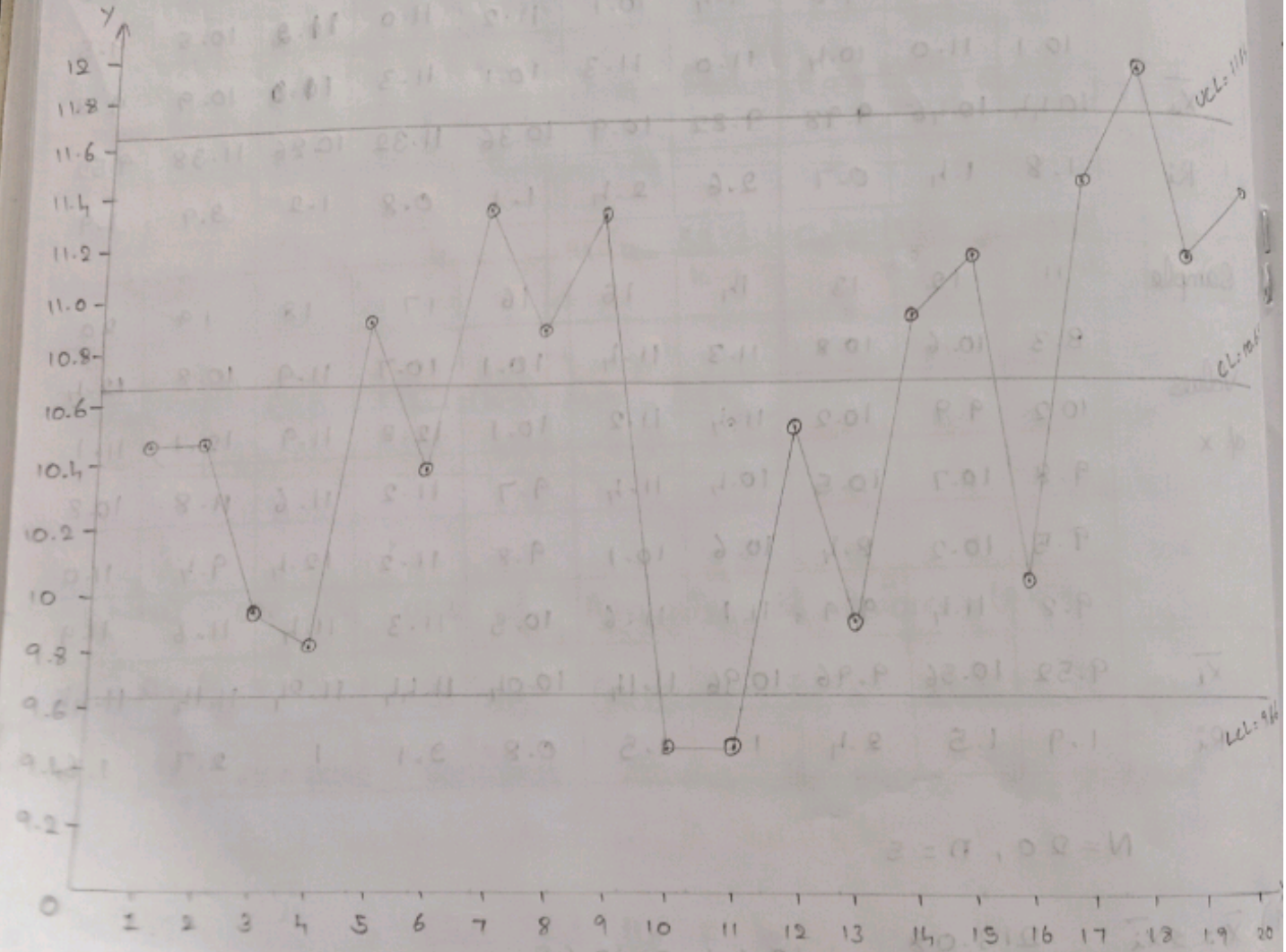
$= (10.65) + (0.577)(1.72) = 10.65 + 0.99$

$= 11.64$

$LCL = \bar{\bar{x}} - A_2 \bar{R}$

$= 10.65 - (0.577)(1.72) = 10.65 - 0.99$

$= 9.66.$



v) Conclusion:

According to $\bar{x}$ - chart, the process is out of control since the some points lies outside of the control lines.

vi) To draw R - chart

$CL = R = 1.72$

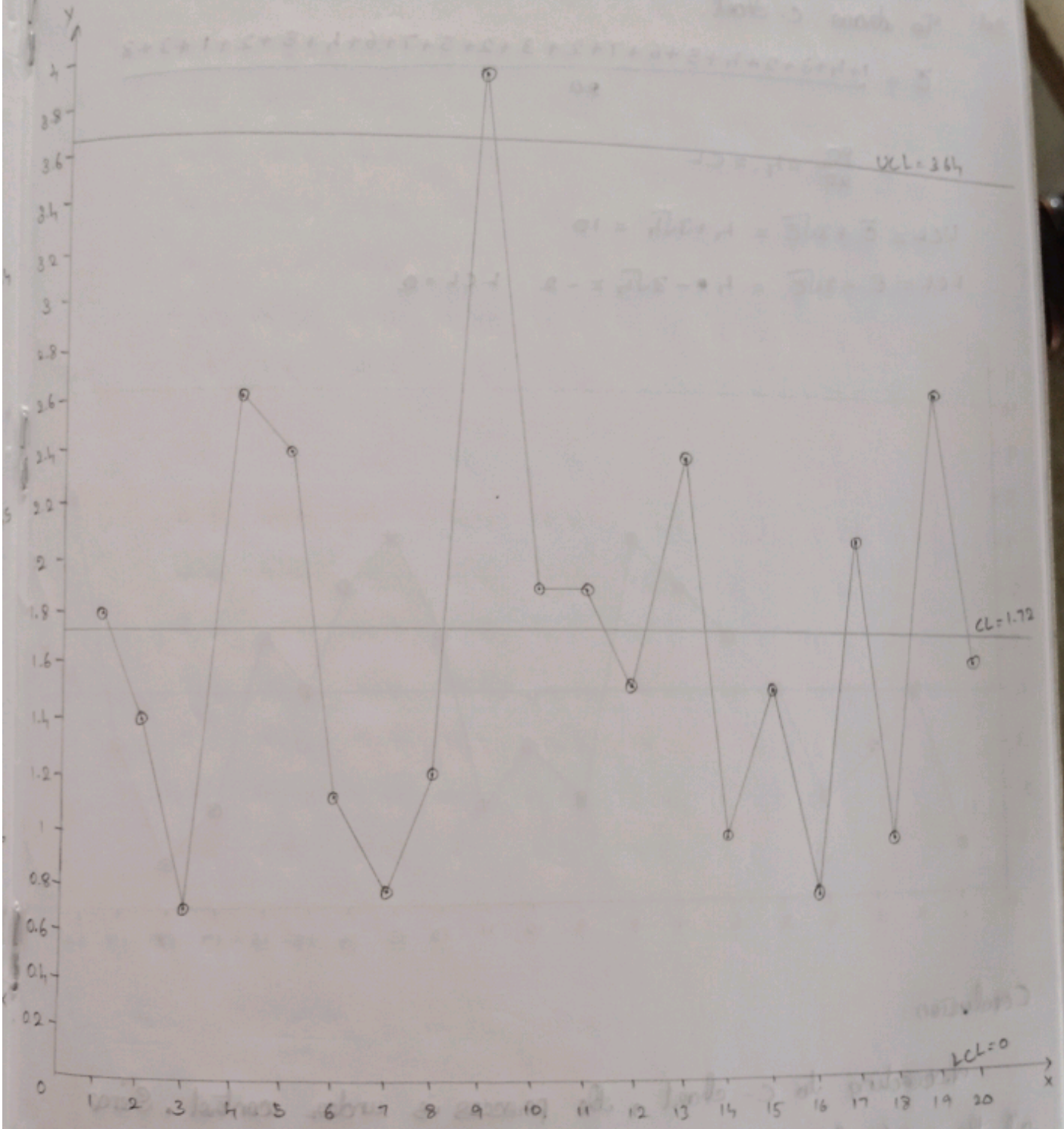
$UCL = D_4 \bar{R}$

$= 2.115 \times 1.72$

$= 3.64$

$LCL = D_3 \bar{R}$

$= 0(1.72) = 0.$



vii) Conclusion:

According to R-chart, the process is out of control, since a point lies outside the control lines.

Tutorial-14

1. 20 pieces of cloth out of different rolls contained respectively 1, 4, 3, 2, 4, 5, 6, 7, 2, 3, 2, 5, 7, 6, 4, 5, 2, 1, 3 and 8 imperfections. Ascertain whether the process is in a state of statistical control.

Sol: To draw C-chart:

$$\bar{C} = \frac{\Sigma C}{N} = \frac{1+4+3+2+4+5+6+7+2+3+2+5+7+6+4+5+2+1+3+8}{20}$$

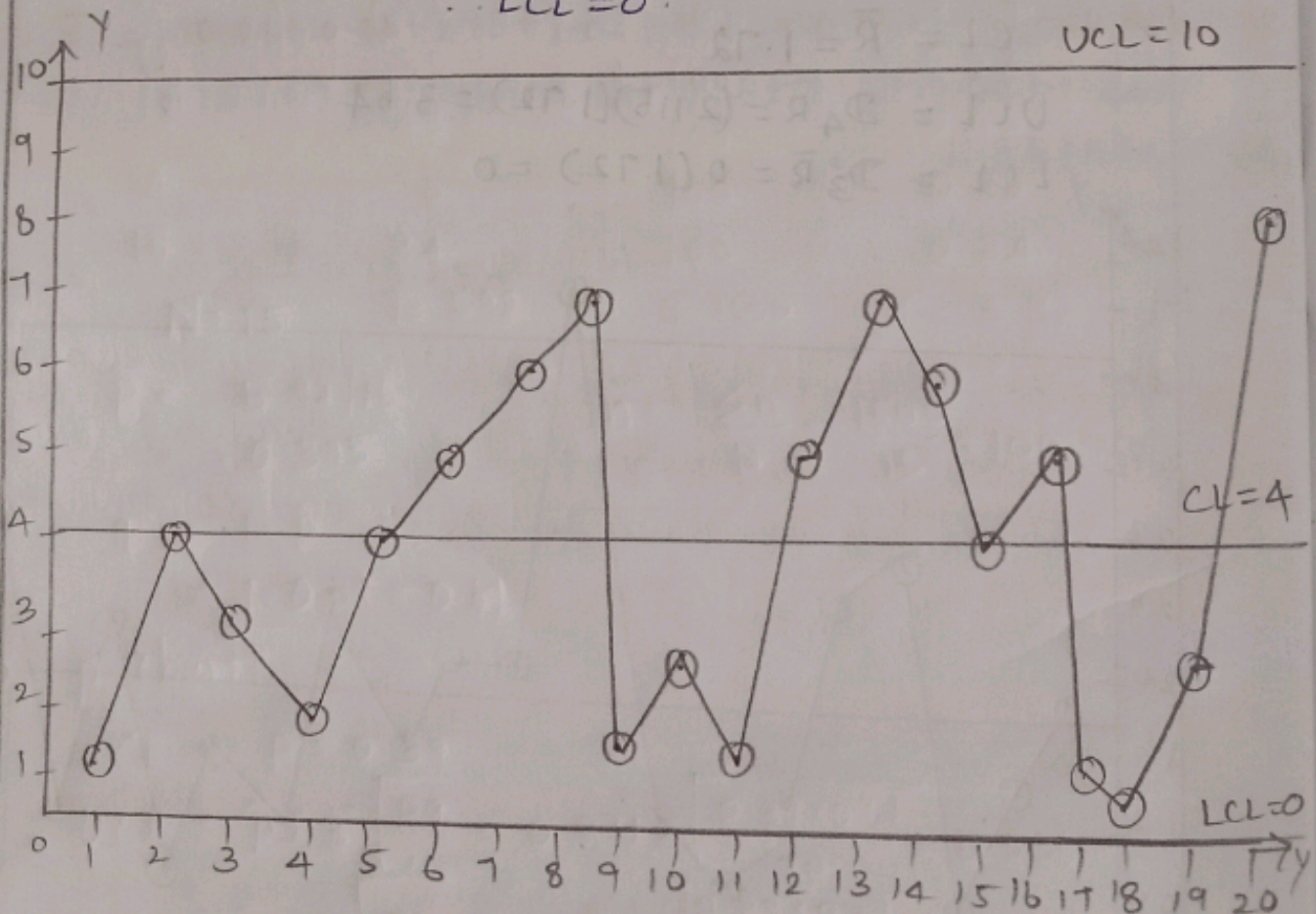
$$= \frac{80}{20} = 4.$$

$$CL = \bar{C} = 4$$

$$UCL = \bar{C} + 3\sqrt{\bar{C}} = 4 + 3\sqrt{4}$$
$$= 4 + 6 = 10$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}} = 4 - 3\sqrt{4}$$
$$= 4 - 6 = -2$$

$\therefore LCL = 0.$



Conclusion: According to c-chart, the process is under control since all the points lie within control limits.

2. The total number of defects in 20 pieces of cloth of equal length is 220. Find the LCL and UCL for c-chart.

Sol: Gn: $\Sigma C = 220$, $N = 20$

$$\therefore \bar{c} = \frac{\Sigma C}{N} = \frac{220}{20} = 11.$$

$$\text{CL} = \bar{c} = 11.$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$
$$= 11 + 3\sqrt{11}$$
$$= 11 + 3(3.32)$$
$$= 11 + 9.96$$
$$= 20.96.$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$
$$= 11 - 3\sqrt{11}$$
$$= 11 - 9.96$$
$$= 1.04$$

$\therefore$ UCL = 20.96, LCL = 1.04.

3. Find the lower and upper control limits for the chart, when $\bar{c} = 6$.

Sol: Gn: $\bar{c} = 6$.

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$
$$= 6 + 3\sqrt{6}$$
$$= 6 + 3(2.45)$$
$$= 6 + 7.35$$
$$= 13.35$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$
$$= 6 - 3\sqrt{6}$$
$$= 6 - 7.35$$
$$= -1.35$$

$\therefore$ UCL = 13.35, LCL = 0.

Tutorial - 15.

1. In an integrated circuit production line, samples of 100 units are checked to electrical specifications on alternate days of month and the results declared as no. of defectives are tabulated below. Draw a p-chart and a np-chart and comment.

Sol:

Sample no	No. of defectives	Sample size	% of defectives
1	24	100	0.24
2	38	100	0.38
3	62	100	0.62
4	34	100	0.34
5	26	100	0.26
6	36	100	0.36
7	38	100	0.38
8	52	100	0.52
9	33	100	0.33
10	44	100	0.44
11	44	100	0.44
12	52	100	0.52
13	45	100	0.45
14	30	100	0.30
15	34	100	0.34
N = 15	T = 592	T = 1500	T = 5.92

$\bar{p} = 5.92/15$

$= 0.395$

$\bar{n} = \frac{\Sigma n}{N} = \frac{1500}{15} = 100$

$\bar{q} = 1 - \bar{p}$

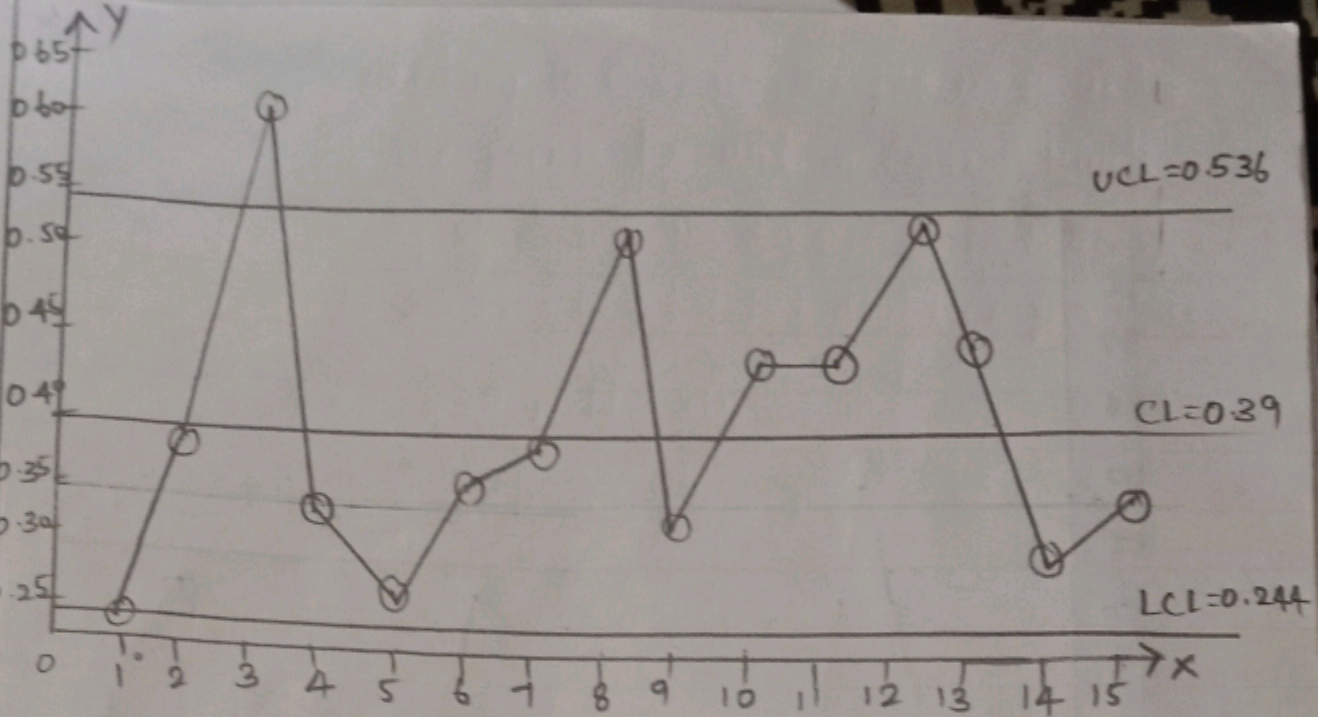
$= 1 - 0.39 = 0.61.$

(i) p-chart:

$CL = \bar{p} = 0.39$

$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{\bar{n}}} = 0.39 + 3\sqrt{\frac{0.39 \times 0.61}{100}} = 0.536.$

$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.39 - 3\sqrt{\frac{0.39 \times 0.61}{100}} = 0.244.$



Conclusion:

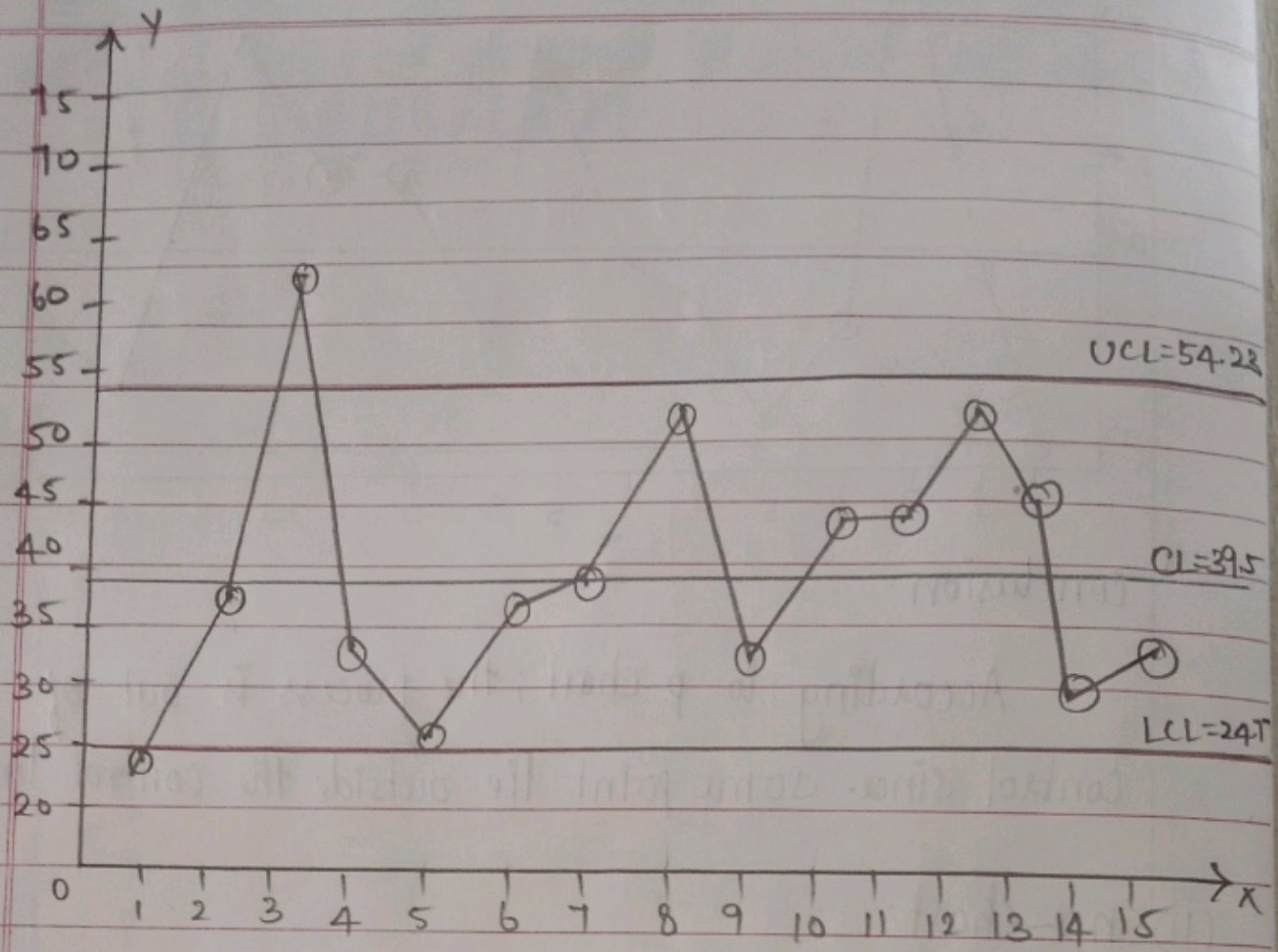
According to p-chart, the process is out of control since some point lie outside the control limits.

(ii) np-chart:

$$CL = \bar{n}\bar{p}$$
$$= 100 \times 0.395$$
$$= 39.5$$

$$UCL = \bar{n}\bar{p} + 3\sqrt{\bar{n}\bar{p}\bar{q}}$$
$$= 39.5 + 3\sqrt{39.5 \times 0.61}$$
$$= 39.5 + 14.73$$
$$= 54.23$$

$$LCL = \bar{n}\bar{p} - 3\sqrt{\bar{n}\bar{p}\bar{q}}$$
$$= 39.5 - 14.73$$
$$= 24.77$$



Conclusion:

According to np-chart, the process is out of control since some points lie outside the control limits.